That's All I know: On the Effectiveness of Logic in Game Theory

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Joint work with Rafael Pass

Admissibility

Admissibility is an old criterion in decision making:

- A strategy σ_i for player i is admissible with respect to a set S of strategies if it is a best response to some *full support* belief of player i: one that gives positive probability to all the strategy profiles S_{-i} for the other players.
- [Pearce '84]: σ_i is admissible iff it is not *weakly dominated*:
  - There is no mixed strategy σ_i' that gives at least as high a payoff to i as σ, no matter what strategy profile in S_{-i} the other players use, and sometimes gives i a strictly higher payoff.

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- [Pearce '84]: σ_i is not strictly dominated iff there is some belief that i could have according to which σ_i is a best response.
  - So the difference between strict dominance and admissibility lies in whether we consider all beliefs, or only beliefs with full support.

Iterated Deletion

It seems natural to ignore strategies that are not admissible.

- We can then iterate, deleting strategies that are are inadmissible with respect to the undeleted strategies.
- This gives us a natural solution concept: *iterated admissibility (IA)* (aka *iterated deletion of weakly dominated strategies*).

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Can similarly consider iterated deletion of *strictly* dominated strategies (ISD).

Common knowledge of rationality characterizes ISD [Brandenburger & Dekel '87; Tan &Werlang '88]

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So how can we characterize IA?

A Puzzle

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Conclusion: common knowledge of admissibility is inconsistent:

- Believing that everyone is playing an admissible strategy (and thus eliminating weakly dominated strategies) is inconsistent with me having a full-support belief.

Our Results

- We provide a new epistemic characterization of IA (and also ISD)
- Key innovation: we assume that not only do players know that other players satisfy appropriate levels of rationality, but that is *all they know*.
  - Specifically, they consider all strategy profiles of other agents consistent with rationality to be possible.

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- We provide a new epistemic characterization of IA (and also ISD)
- Key innovation: we assume that not only do players know that other players satisfy appropriate levels of rationality, but that is *all they know*.
  - Specifically, they consider all strategy profiles of other agents consistent with rationality to be possible.
- Essentially the same formula characterizes both IA and ISD
  - The only difference is the (logical) *language* used by agents to reason about each other.
- Conclusion: Knowledge is good, but we must know our limitations :-)

Final thoughts

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Human behavior is a mystery
But Moshe says that logic sheds some light
And all I know is that he's often right
So I'll do research more logically

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Thanks, Moshe, for years of friendship and exciting collaborations, and for being an inspiration to us all.