An Automata-Theoretic Approach to . . . .

Mateo Perez, Fabio Somenzi, Ashutosh Trivedi

University of Colorado Boulder

August 1, 2022

Joint work with Ernst Moritz Hahn, Sven Schewe, and Dominik Wojtczak

An Automata-Theoretic Approach to Reinforcement Learning

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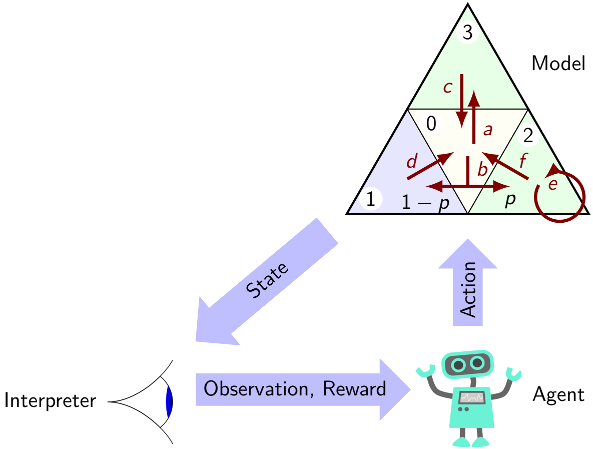
Reinforcement Learning



The problem

Specifying objectives via reward simplifies the development of new algorithms.
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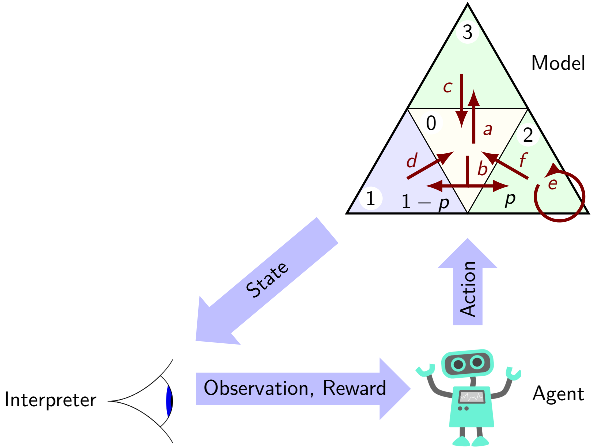
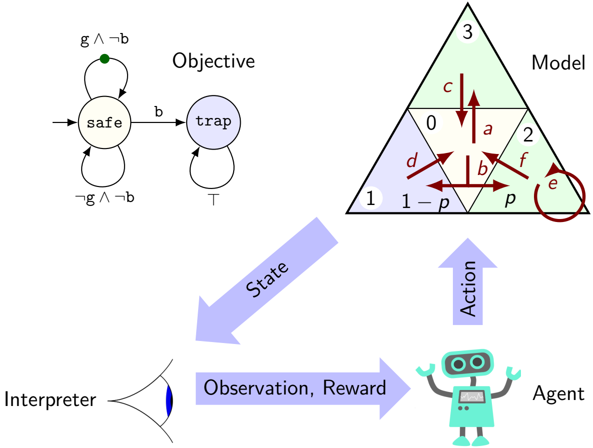
Let's specify a formal requirement and have it "compiled" to the representation used by RL.
We can use Linear Temporal Logic and ideas from probabilistic model checking!

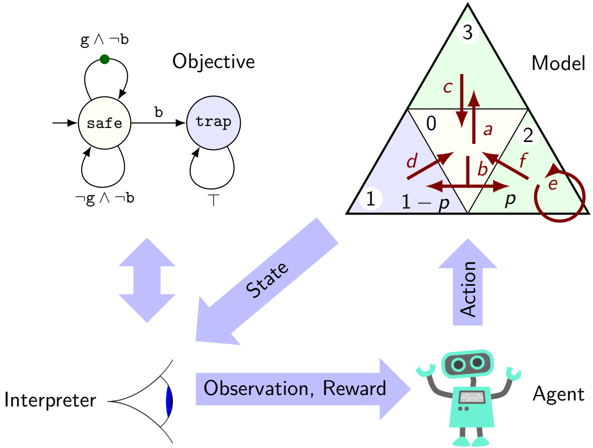
Model-free reward translation



Model-free reward translation



Model-free reward translation



Objective

Model

State

Action

Observation, Reward

Interpreter

Agent

Rabin to discounted reward[1]

Can we use Rabin automata? No correct translation has been proposed.

[1] An Impossibility Result in Automata-Theoretic Reinforcement Learning. ATVA 2022.

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Optimal strategies in RL mix.

$$Q^*(s, a_0) = 5, Q^*(s, a_1) = 5, Q^*(s, a_2) = 3$$

Any strategy that mixes a_0 and a_1 in s maintains optimality.

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Optimal strategies for Rabin may not mix!

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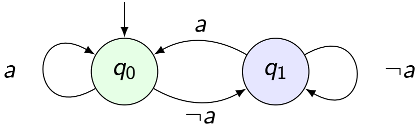
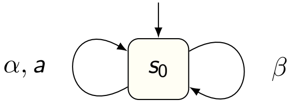
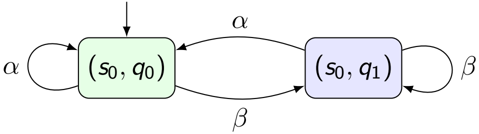
Rabin to discounted reward

$\mathbf{FG}a \vee \mathbf{FG}\neg a$





$\{\langle q_0, q_1\rangle, \langle q_1, q_0\rangle\}$



α is optimal, β is optimal, but mixing α and β is not.

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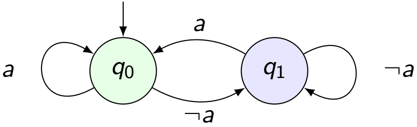

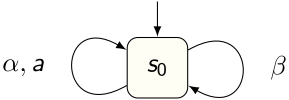

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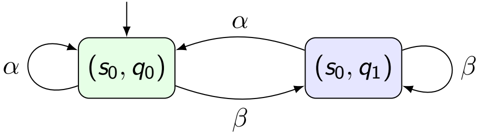

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We can not reduce a Rabin automaton directly to reward without additional memory.

Büchi to discounted reward

To use Büchi automata, we may require nondeterminism.

[1] Automatic Verification of Probabilistic Concurrent Finite-State Programs. Moshe Y. Vardi. FOCS 1985.

[2] Limit-Deterministic Büchi Automata for Linear Temporal Logic. Sickert et al. 2016

[3] Good-for-MDPs Automata for Probabilistic Analysis and Reinforcement Learning. TACAS 2020

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For an automata-theoretic approach to model-checking of probabilistic programs "we eliminate the need for a complete determinization of the given automaton."[1] – Moshe Vardi

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We can use suitable limit-deterministic Büchi automata[2] and more generally Good-for-MDPs (GFM) automata.[3]

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GFM Büchi to discounted reward[1]

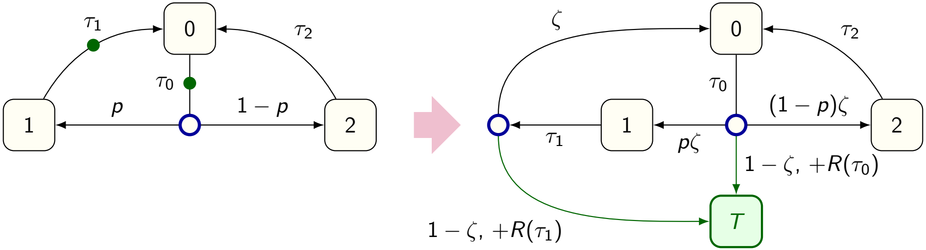
How do we assign the reward?

- $+1$ reward on accepting edges and 0 otherwise does not work. Why?
- maximize expected frequency of accepting edges $\neq$ maximize probability that the frequency is positive
- Seeing accepting edges on every other step with probability 1 is valued lower than seeing accepting edges on every step with probability $2/3$.

[1] Omega-Regular Objectives in Model-Free Reinforcement Learning. TACAS 2019

GFM Büchi to discounted reward

- Instead, let's introduce an additional parameter $\zeta \in (0,1)$.
- On accepting edges with probability $1-\zeta$ assign $+1$ reward and terminate.



- Under total reward, satisfying traces are given a value of 1.
- Under total reward, traces that are not satisfying are given a value of ε with $\lim_{\zeta\uparrow 1}\varepsilon = 0$.

GFM Büchi to discounted reward

Theorem (Limit reachability)

For a given MDP, there exists a threshold for $\zeta' \in (0,1)$ *and for* $\gamma' \in (0,1)$ *such that for any* $\zeta > \zeta'$ *and* $\gamma > \gamma'$ *maximizing the discounted reward from the construction above maximizes the probability of satisfaction of the Büchi objective.*

Summary

Instead of assigning reward manually, perform a translation from a high-level objective.
For omega-regular objectives (LTL):

[1] Model-Free Reinforcement Learning for Stochastic Parity Games. CONCUR 2020.
[2] Model-Free Reinforcement Learning for Lexicographic Omega-Regular Objectives. FM 2021.

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- Parity[1]: Needed for games. Have a set of increasingly weighted coins with accepting and rejecting sinks.

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- Parity[1]: Needed for games. Have a set of increasingly weighted coins with accepting and rejecting sinks.
- Lexicographic[2]: Add a memory gadget. Then, use large enough weights to separate the associated Büchi rewards.

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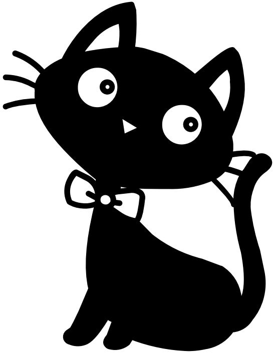
Mungojerrie[1]



MUNGOJERRIE

Formal Reinforcement Learning

https://plv.colorado.edu/mungojerrie/

[1] Mungojerrie: Reinforcement Learning of Linear-Time Objectives. Preprint 2021

Thank you!